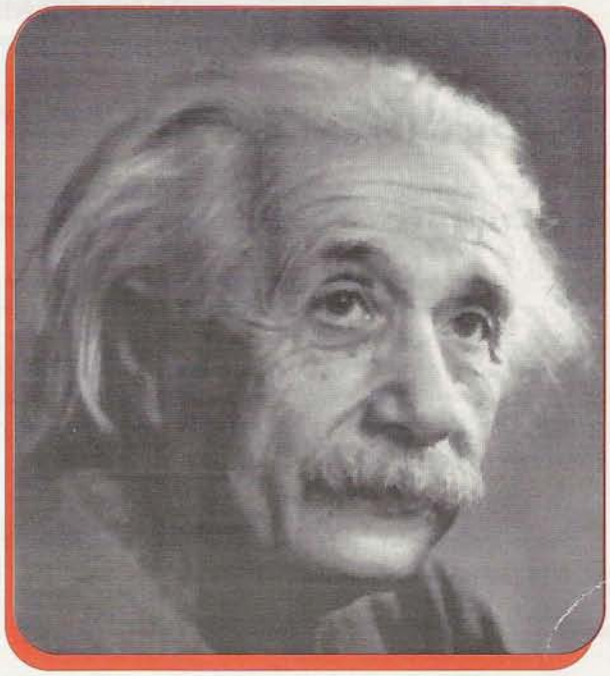
You Don't Have to Be Einstein... to Understand Decibels



Decibels are handy for measuring and comparing all sort of things, from antenna gain to RF power. Here's how to make all the numbers add up (using a scientific calculator).

By Ron Hranac, NØIVN*

Editor's Note: If you flunked high school math, this article may be hazardous to your health. But if you passed, it should help you learn how to make use of the various variations on the decibel, such as dBd, dBi, dBm and dBW, in real-life calculations.

We hams use a wide variety of measurement units, from farads and henrys to ohms and volts. Transmitter power, for example, is usually measured in watts, while receiver sensitivity is generally measured in microvolts (μV), and any mismatch between your transmitter and your antenna is expressed in SWR.

Using different units makes it difficult to combine and compare the different elements that go into how well a signal will be heard over a given path. Wouldn't it be handy to have a single unit of measurement with which you could calculate the performance of all these elements and work out your overall system performance? Well, there is such a unit—the ubiquitous decibel (dB).

You're probably aware already that gain and loss in your antenna system is often expressed in dB. But it's also possible to measure transmitter power and receiver gain in terms of dBm, or decibel-milliwatts. When everything's measured in dB, it then becomes easier to add and subtract values—adding apples and apples instead of apples and oranges.

This article is about the mathematics of the decibel as it's used in amateur radio and is adapted from articles I've previously written for *Communications Technology*, *International Cable* and *SPEC-COM* magazines, and the *Rocky Mountain VHF+* newsletter. Don't let your eyes glaze over just yet, though. I promise I'll try to keep this at a reasonably down-to-earth level, and provide some useful information while I'm at it. Grab a fresh cup of coffee and your scientific calculator, and tag along for some fun with the decibel.

A Little History

It's best to think of the decibel as a shorthand method of dealing with numbers. By itself, the decibel can only express a ratio between two values, although when coupled to a reference, it can express absolute values. Even in the latter case, the expression is actually a ratio of some value to a reference.

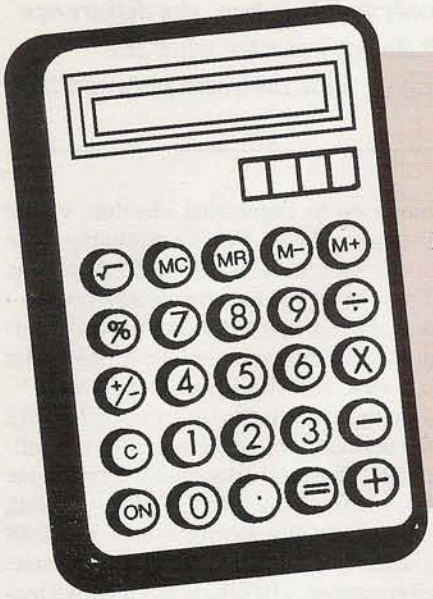
"To fully understand decibels, it helps to also know a little about logarithms....In this case, think of logarithms as the building blocks of the decibel."

The decibel had its origins in the telephone industry. In the early part of the century, telephone engineers discovered that a 10-mile length of a certain type of telephone cable attenuated the power of a signal by a fixed amount. They named that amount of power loss the *Bel*, in honor of Alexander Graham Bell. Mathematically, the loss was determined to be the base 10 logarithm of the ratio of the telephone cable's output power to its input power:

$$\text{bel} = \log (P_1/P_2)$$

where P_1 was the signal power at the input of the telephone cable and P_2 was the signal power at the cable's output. But like the Farad, which measures capacitance, the Bel is very cumbersome to deal

**Ron Hranac, NØIVN, is a VHF contester from Colorado...and that extreme rarity—someone who actually understands, and can explain, decibels!*



$$\text{decibel} = 10 \times \log (P_1/P_2)$$

$$\text{bel} = \log (P_1/P_2)$$

$$L_{FS} = 36.6 + [20 \times \log (F)] + [20 \times \log (D)]$$

$$\text{dBW} = 10 \times \log (\text{watt})$$

$$\text{dBm} = 10 \times \log (\text{milliwatts})$$

with—especially on shorter lengths of cable—so the telephone folks divided everything by 10 and came up with the *decibel*, which is $^1/_{10}$ of a Bel. The decibel could then be described mathematically as:

$$\text{decibel} = 10 \times \log (P_1/P_2)$$

Some Basics First

I need to backtrack for just a moment. To fully understand decibels, it helps to also know a little about logarithms. Like the decibel, logarithms are a fundamental part of the mathematics of amateur radio. In this case, think of logarithms as the building blocks of the decibel. Logarithms also can be thought of as a type of mathematical shorthand that can be used to deal with numbers, especially very small or very large numbers. Furthermore, they can be used to reduce multiplication and division to much simpler addition and subtraction.

Consider the following representations of the numbers 0.001, 100 and 10,000:

$0.001 = 1/(10 \times 10 \times 10) = 10^{-3}$
$100 = 10 \times 10 = 10^2$
$10{,}000 = 10 \times 10 \times 10 \times 10 = 10^4$

These three numbers have something in common. Each can be represented as 10 raised to some power. The power to which 10 is raised in each case is called an exponent or logarithm. Since the number 10 is being raised to some power in the previous examples, we can say that the exponents or logarithms are all base 10 logarithms, or simply $\log_{10}$. Base 10 logarithms are often abbreviated as *log*.

In the world of base 10 logarithms, we can simplify even more by dropping the base 10 and working with only the exponents (logarithms). Thus, the base 10 logarithm, or log, of 0.001 is –3; the log of 100 is 2; and the log of 10,000 is 4. Each of these also could be written as

log (0.001) = -3
log (100) = 2
log (10,000) = 4

From this, it should be apparent that log (1,000) = 3 and log (100,000) = 5, since $1{,}000 = 10^3$ and $100{,}000 = 10^5$.

But what is, say, log (495)? Well, it should be some number between 2 and 3, because log (100) = 2 and log (1,000) = 3. Using either a logarithm table (if you remember those, you're dating yourself) or a much more convenient scientific calculator, you'll find that log (495) = 2.6946. That is, $495 = 10^{2.6946}$.

I mentioned that logarithms can be used to reduce multiplication and division to addition and subtraction. Here's how. If you were to multiply 100,000 x 1,000, your first choice is to do it the old-fashioned way:

```
  100,000
    1,000
   000000
  000000
 000000
100000
100000000
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Or, you can do it a much easier way: Add the logarithms of the two original numbers! The sum of the two logarithms will be the exponent (logarithm) of the answer. We know that log (1,000) = 3 and log (100,000) = 5, so just add 3 + 5. We get 8, so the answer to the multiplication problem doing it this way is 10^8, or 100,000,000. If you want to divide numbers, just subtract their logarithms. Pretty simple, huh?

"Since the decibel expresses a ratio between two power levels, it's ideal for describing gain and loss. In fact, comparing a device's output power to its input power is one common use for the decibel."

Decibels and Power Levels

Now, let's go over some practical examples of using decibels to compare different RF power levels. Earlier I showed you the formula that mathematically describes the decibel:

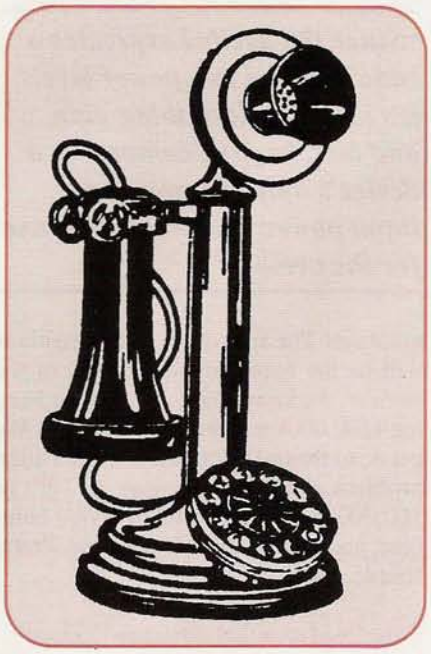
$$\text{dB} = 10 \times \log (P_1/P_2)$$

Since the decibel expresses a ratio between two power levels, it's ideal for describing gain and loss. In fact, comparing a device's output power to its input power is one common use for the decibel. For example, it's a whole lot easier to say something like "the loss through a 50-foot piece of coax is 3 dB" instead of "the loss through a 50-foot piece of coax is 50 watts if the input is 100 watts", or "...1 watt if the input is 2 watts," etc.

Let's work through an example: Suppose your VHF "brick" amplifier has an output of 100 watts, and your friend's barefoot rig has an output of 50 watts. How much more power does your amplifier produce, in decibels, than your friend's rig? Here's the solution, using the previous formula:

$\text{dB} = 10 \times \log (P_1/P_2)$
$\text{dB} = 10 \times \log (100/50)$
$\text{dB} = 10 \times \log (2)$
dB = 10 x 0.3010 (using a scientific calculator or log table)
dB = 3.01

Your amp has 3 dB more output power than your friend's rig (well, OK, 3.01 dB). The important thing to understand from this example is that whenever the ratio of any two power levels is *two*, the difference between them will always be 3 dB.



The decibel is one-tenth of a Bel, a unit of measurement named after telephone pioneer Alexander Graham Bell. It was first used by telephone engineers to calculate loss in telephone cables.

"...whenever the ratio of any two power levels is two, the difference between them will always be 3 dB. It doesn't matter what the original power levels are; we are dealing with the ratio between the two levels."

It doesn't matter what the original power levels are; we're dealing with the *ratio* between the two levels. If one station's output power is 1,500 watts and another's is 750 watts, if one is 10 watts and the other is five watts, or if one is 20 milliwatts and the other is 10 milliwatts, the difference will always be 3 dB.

Look at it another way: If you have a 100-foot piece of coax with 3 dB of loss at 144.220 MHz, the power at the cable's output end will always be half as much as the power at the cable's input end (assuming no impedance mismatches or VSWR problems). If the input to the coax is 100 watts, the output at the other end will be 50 watts; if the input is 10 watts, the output will be 5 watts, and so on.

Here are a couple rules of thumb to remember: A 3-dB change (actually 3.01 dB) means that the power level has either doubled or been reduced by half. A change of 10 dB means that the power level has increased 10 times or has been reduced to $^1/_{10}$ of its original value. You can use these two rules to "guesstimate" simple power gains and losses.

Making Guesstimates

For example, if you have a brick with 10 dB of gain, what will the output power be if the input is 10 watts? Simple: 100 watts (10 dB means the power level will change by a factor of 10. Thus, 10 watts x 10 = 100 watts). What if the amplifier has 13 dB of gain? Still simple: 200 watts. How do we get that figure? The first 10 dB will boost your 10 watts to 100 watts, and another 3 dB will double that to 200 watts (10 dB + 3 dB = 13 dB).

What if the amplifier's gain is 7 dB? That's right: 50 watts! Here's how to figure this one in your head: 10 dB will increase 10 watts to 100 watts; *subtracting* 3 dB from the 10 dB (10 dB - 3 dB = 7 dB) means the 100 watts will then be reduced by half to 50 watts. Keep in mind that I'm taking some journalistic liberties here and anytime I refer to 3 dB or increments of 3 dB, I actually mean 3.01 dB.

Let's go through a couple more examples. What is the gain of a linear amplifier that requires 75 watts of drive to produce 1500 watts output?

$dB = 10 \times \log (P_1/P_2)$
$dB = 10 \times \log (1{,}500/75)$
$dB = 10 \times \log (20)$
$dB = 10 \times 1.3010$
$dB = 13.01$

If you measure 150 watts at the output of your VHF brick amplifier in the shack and 110 watts at the other end of the coax feeding the antenna, what is the loss through the coax?

$dB = 10 \times \log (P_1/P_2)$
$dB = 10 \times \log (150/110)$
$dB = 10 \times \log (1.36)$
$dB = 10 \times 0.1347$
$dB = 1.35$

Most S meters are calibrated such that each S unit is about 6 dB, so a loss of 1.35 dB between your shack and antenna will barely be noticeable—if at all—to the station at the other end of your QSO. That amount of loss represents less than $^1/_4$ of an S unit. Unless you're doing very weak signal EME or something similar, this is a negligible amount.

Decibels and Absolute Values

Now that we've covered power ratios and their relationship to decibels, let's move on to expressing absolute values using decibels. You've probably seen terms such as +30 dBm, -10 dBm, etc. They refer to discrete power levels—actually, ratios to a defined "zero dB" reference. In the examples above, +30 dBm = 1 watt, and -10 dBm = 0.0001 watt.

In the case of *dBm (decibel-milliwatt)*, the defined "zero dB" reference is 1 milliwatt. That is, 0 dBm equals 1 milliwatt. So, when we say that +30 dBm = 1 watt, we are actually saying 1 watt is 30 dB *greater* than the 1 milliwatt reference. (Remember, +10 dB always means a tenfold increase, so +30 dB = 10+10+10, or 1,000 times the original value.) Likewise, -10 dBm means that 0.0001 watt is 10 dB *less* than the 1 milliwatt reference. See? Even when dealing with absolute values, decibels are still nothing more than ratios between two values!

You can convert from milliwatts to dBm using the formula:

dBm = 10 x log (milliwatts)

Here's an example. Your 10 GHz transverter's output is 100 milliwatts. What is that level in dBm?

$dBm = 10 \times \log (\text{milliwatts})$
$dBm = 10 \times \log (100)$
$dBm = 10 \times 2$
$dBm = 2$

In this case, the answer is +20 dBm. Technically, this means 100 milliwatts is 20 dB greater than the 1 milliwatt reference. Another way to figure this is with the rules of thumb I showed you previously (a 3 dB change means we change the original amount by a factor of two; a 10 dB change means we change the original amount by a factor of 10). If 0 dBm = 1 milliwatt, then increasing 1 milliwatt by 10 dB would give us 10 milliwatts (1 milliwatt x 10 = 10 milliwatts). Another 10-dB increase would change the 10 milliwatts to 100 milliwatts (10 milliwatts x 10 = 100 milliwatts). Or, 0 dBm + 10 dB + 10 dB = 20 dBm.

What if the original power level is in watts instead of milliwatts? Well, you can use the same formula if you first convert

the amount in watts to milliwatts. You do this by multiplying the amount in watts by 1,000. For instance, 3 watts is 3,000 milliwatts (3 x 1,000 = 3000 milliwatts).

Now, let's assume your 2-meter rig's output power is 25 watts. What is its output in dBm? First, convert 25 watts to milliwatts by multiplying by 1,000 (25 x 1,000 = 25000 milliwatts). Then use the previous formula to determine the power in dBm.

dBm = 10 x log (milliwatts)
dBm = 10 x log (25,000)
dBm = 10 x 4.3979
dBm = 43.98

About Face!

To go the other direction and convert from dBm to milliwatts, use one of the following formulas (they actually are the same formula; one is just written a little differently than the other). I'm showing both, because some calculators have an <INV LOG> key, and others perform the same function using a <10^X> key.

milliwatts = <INV LOG>(dBm/10)
or
milliwatts = $10^{(dBm/10)}$

Let's suppose that you just bought a 1296-MHz transverter, and the instructions say the required drive from a 432.100 MHz IF rig is +13 dBm. What is that drive level in milliwatts? Here's how to find the answer:

mW = $10^{(dBm/10)}$
mW = $10^{(13/10)}$
mW = $10^{(1.3)}$
mW = 19.95

If you want to convert milliwatts to watts, just divide the amount in milliwatts by 1,000. For the previous example, 19.95 mW = 0.02 watt (19.95/1,000 = 0.0195, rounded to 0.02 watt).

And Now, the dBW

Before moving on, I'd like to review one other decibel conversion. It's the *dBW*, or *decibel-watt*. Here, the "zero dB" reference is 1 watt. That is, 0 dBW = 1 watt. While not as common as the previously discussed conversions, the dBW does occasionally show up in amateur radio. Here's the formula to convert from watts to dBW:

dBW = 10 x log (watt)

A quick example: Most HF rigs have a nominal maximum power output of 100 watts. What is that output power expressed in dBW?

dBW = 10 x log (watt)
dBW = 10 x log (100)
dBW = 10 x 2
dBW = 20

That's pretty straightforward. To convert from dBW to watts, use one of the following formulas:

watts = <INV LOG>(dBW/10)
or
watts = $10^{(dBW/10)}$

Try an example on your own using one of these last formulas. The process is similar to converting from dBm to milliwatts.

Putting It All Together

Now let's work through a real-life situation that puts together everything that's been discussed so far. Our task is going to be to calculate the received signal strength at the input to a friend's transceiver in both dBm and microvolts μV, given some assumptions about the transmitting and receiving stations.

Let's say you're having a CW QSO with a friend who lives 110 miles away. Your 432.100-MHz transceiver output is 25 watts, and you've got 60 feet of RG-213 cable (4.8-dB loss per 100 feet at 432.1 MHz) between the output of your rig and the antenna. Your antenna is a 16-element Yagi, which has a published gain of 14.0 dBd (16.14 dBi)†. Your friend has a similar setup, except his antenna is a 10-element Yagi with 11.5 dBd (13.64 dBi) of gain. He, too, has 60 feet of feedline, but he recently upgraded to Belden 9913 coax (2.9-dB loss per 100 feet at 432.1 MHz). Assuming an unobstructed line-of-sight path between the two stations and no unusual propagation conditions, what is the received signal strength—both in dBm and in μV—at the input to your friend's rig?

Here's the formula we're going to use to calculate the receiver input in dBm:

$$P_R = P_T - F_T + A_T - L_{FS} + A_R - F_R$$

where
P_R = receiver input, dBm
P_T = transmitter output power, dBm
F_T = transmit feedline loss, dB
A_T = transmit antenna gain, dBi
L_{FS} = free space path loss between the two stations, dB
A_R = receive antenna gain, dBi, and
F_R = receive feedline loss, dB.

One of the first things you'll notice is that all of this formula's units are in some variation of the decibel. This makes things a whole lot easier! However, before using the formula we have to convert the transmitter output power, which is in watts, to dBm.

To do this, as we explained a few paragraphs back, we'll use the formula:

dBm = 10 x log (milliwatts)

Milliwatts? But the transmitter output is shown in watts! No problem, just remember to multiply the transmitter's output power in watts by 1,000; so 25 watts x 1,000 = 25000 milliwatts. Let's start plugging in numbers.

dBm = 10 x log (25000)
dBm = 10 x 4.3979
dBm = +43.98

Looking back at the formula to calculate receiver input, there's one other term that must be dealt with. It's L_{FS}, or free space path loss. This is the amount of signal attenuation that will occur "through the air" between two locations. The next formula can be used to calculate L_{FS}, but it won't take into account additional loss that is the result of blockage by trees, buildings, local terrain, and other obstructions. It also doesn't account for signal enhancement due to unusual propagation conditions, or signal variations caused by reflection or refraction. In real-world situations, one or more of these would likely have some effect on the transmitted signal, possibly making the actual received signal level quite a bit different from what we calculate. That said, here's the formula for calculating free space path loss:

$$L_{FS} = 36.6 + [20 \times \log (F)] + [20 \times \log (D)]$$

where
L_{FS} = free space path loss, dB
F = frequency, MHz
D = distance, statute miles

So, how much will the signal be attenuated over the 110-mile path between

† A dipole antenna has 2.14-dB gain over a theoretical *isotropic* antenna, which radiates equally in all directions. Therefore, 0 dBd, the gain of a dipole in free space, equals +2.14 dBi, and any real-life antenna will always have its gain in dBi appear 2.14 dB greater (sometimes 2.15) than its gain in dBd. The important thing is to compare dBd to dBd and dBi to dBi.

your transmit antenna and your friend's receive antenna?

L_{FS} = 36.6 + [20 x log (432.1)] + [20 x log (110)]
L_{FS} = 36.6 + [20 x 2.6356] + [20 x 2.0414]
L_{FS} = 36.6 + 52.7117 + 40.8279
L_{FS} = 130.14 dB

Now, we need to figure out coax attenuation for both stations. The RG-213 at your station has 4.8 dB of loss per 100 feet at 432.1 MHz, but you have only 60 feet of it. Therefore, your feedline's loss is 4.8 x 0.60 = 2.88 dB. Your friend is using 9913, which has 2.9 dB of loss per 100 feet at 432.1 MHz. He, too, has only 60 feet of feedline, so his coax attenuation is 2.9 x 0.60 = 1.74 dB. (As a side note, if he had 150 feet of 9913 feedline, the loss would be 2.9 x 1.50, or 4.35 dB.)

Now we're ready to plug everything in:

$P_R = P_T - F_T + A_T - L_{FS} + A_R - F_R$
P_R = 43.98 - 2.88 + 16.14 - 130.14 + 13.64 - 1.74
P_R = -61.0 dBm

"In the case of dBm (decibel-milliwatt), the defined 'zero dB' reference is 1 milliwatt. That is, 0 dBm equals 1 milliwatt."

The signal strength at the input to your friend's receiver is -61 dBm. That's quite a bit weaker than the +43.98 dBm (25 watts) at the output of your transmitter—in fact, it's nearly 105 dB less!

Moving on to Microvolts

We've figured out one answer: the receiver input in dBm. The next step is to figure out the receiver input in µV. To do this, we first have to convert the receiver's dBm input to milliwatts, using the following formula:

$mW = 10^{(dBm/10)}$
$mW = 10^{(-61/10)}$
$mW = 10^{(-6.1)}$
mW = 0.000000794328

Next, change from milliwatts to watts by dividing by 1,000: 0.000000794328/1,000 = 0.000000000794328 watt. Pretty small signal, huh? Now, using a variation on Ohm's Law ($E = \sqrt{PR}$), we can change the signal level from watts to volts, and then to microvolts. When doing this, we assume that the entire receive system impedance ("R" in the Ohm's Law formula) is 50 ohms.

$E = (\sqrt{0.000000000794328 \times 50})$
$E = \sqrt{0.0000000397164}$
E = 0.0001993 volt

Still a pretty small signal, huh? Well, maybe not as small as it seems. To convert volts to microvolts, just multiply the amount in volts by 1,000,000: 0.0001993 x 1,000,000 = 199.29 µV. Considering that most contemporary amateur transceivers have a receive sensitivity spec better than 1 µV (less than 0.2 µV is typical), this calculated received signal strength is actually pretty decent.

To put it in perspective, when the ARRL published its test results of the Yaesu FT-736R back in the May 1990 issue of *QST*, the '736 had a 432-MHz S-meter sensitivity of 5.2 µV for S9. Our 199.29-µV signal would be about 30 dB over S9.

Remember, this calculated signal level assumes ideal conditions and no obstructions (including the good ol' Earth) between you and your friend.

Class dismissed! ■

6-Meter Internet Contest Results

Following are the top scorers in the first Internet 6-Meter Contest, as reported by contest originator Ken Ramirez, N4UK. The contest coincided with the ARRL June VHF Contest.

USA/VE
Single Operator/High Power

1st	K5AM	13,575	Plaque winner
2nd	VE1ASJ	9,639	Certificate winner
3rd	WZ8D	5,625	Certificate winner
4th	KB5IUA	5,508	Certificate winner
5th	KØFF	3,267	Certificate winner

QRP

1st	CG2PIJ	5,289	Plaque winner
2nd	AA4S	884	Certificate winner

Multi-operator

1st	W2DRZ	31,195	Plaque winner
2nd	WS4F	20,732	Certificate winner
3rd	KØYO	4,050	Certificate winner

Rover

1st	K8WW	9,360	Plaque and KB6KQ miniloop winner
2nd	AA4R/8	1,896	Certificate winner

DX
Single operator

1st	GØAEV	28,130	Plaque winner
2nd	G1IOV	8,845	Certificate winner
3rd	CO2OJ	120	Certificate winner

Highest Multiplier Total	GØAEV	24 DXCC countries + 73 Grids = 97 mults.
Most Grids worked	W2DRZ	83 Grids
Most Grids worked by single op.	K5AM	73 grids
Most Grid fields worked	GØAEV and K5AM	8 Grid Fields

K7XC will be keeping the 6-Meter Internet DX Contest records. There were plenty of certificates and plaques that went unclaimed this year. Keep the contest in mind next year!

—N4UK